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RESEARCH ARTICLE

Gerechte-Based Mating-Environmental Designs for Varietal Improvement

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Abstract

Diallel crosses are important mating designs used to study and gain useful information to enhance genetic traits in crops. Due to the scarcity of resource availability, one can use a fraction of these crosses, *i.e.*, partial diallel crosses (PDCs), which enlarges the applicability of mating designs. A more suitable design for breeding experiments is mating-environmental design, or ME design, which can be used both as a mating and an environmental design. It is interesting to explore the possibility of obtaining efficient ME designs from Gerechte designs (possessing excellent mathematical properties) as base designs. Gerechte design is an extension of a conventional Latin square design, which has every symbol/treatment in each row, column, and region at most once. In this study, three construction methods for diallel cross experiments have been developed that yield highly efficient designs for a wide range of parametric combinations. An R package is also developed for the generation, categorization, and analysis of proposed designs to enhance the application potential.

Keywords: Breeding experiments, Canonical efficiency factor, Diallel cross, R package (MEDesigns).

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Introduction

In genetic research, breeding experiments are fundamental that aim to improve inherent traits such as productivity, quality, disease resistance, and environmental adaptability. Mating designs like diallel crosses are widely utilized to explore genetic variation among multiple parental lines and the identification of important traits and gene interactions. Through the proper conduct of systematic mating of different lines and analysis of the resulting offspring, researchers can obtain valuable insights into heritability and genetic potential. These methods not only help in developing high-yielding, resilient crops but also make a huge contribution to sustainable agricultural practices and food security, addressing the challenges posed by climate change and a growing global population.

Due to resource limitations, the most commonly used mating designs are partial diallel crosses (PDCs), which are a fraction taken from all possible diallel crosses. Furthermore, another important genre of designs can be obtained through merging these mating designs with environmental designs (block, row-column, *etc.*) for laying out the selected crosses in environmental conditions, which is called mating environmental designs or ME designs. It is always advantageous for a breeder to opt for an ME design that refers to a single design used for the formation of sample crosses and laying them out in environmental conditions. Hinkelmann and Kempthorne (1963) considered the correspondence between PDCs and partially balanced incomplete block (PBIB) designs with *m*-associate classes. These designs are very useful to study the indicators of breeding

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experiments, *i.e.*, general combining ability (GCA) and specific combining ability (SCA). Here, GCA is an indicator of how well a particular parent generally combines with other parents to produce a superior offspring. On the other hand, sca refers to the performance of a particular cross between two specific parental lines that is different from what would be expected based on their GCA. For details one can refer to Griffing (1956), Kempthorne and Curnow (1961), Das and Sivaram (1968), Narain *et al.* (1974), Mathur and Narain (1976), Arya and Narain (1977), Narain and Arya (1981), Arya (1983), Agarwal (1985), Kaushik and Puri (1989), Ghosh and Divecha (1997), Gupta and Kageyama (1994), Parsad *et al*. (1999), Bhar and Gupta (2002), Varghese and Varghese (2017) Varghese *et al*. (2005) and Harun *et al*. (2024).

Regions of Gerechte designs can be explored to construct efficient ME designs involving diallel cross experiments.

Gerechte designs are a generalization of popular Sudoku designs where each symbol appears in each row, column, and region only once, and the absence of some of the symbols in rows, columns, or regions leads to an incomplete Gerechte design. For detailed information, one can see Bailey *et al*. (1990, 1991, 2008), Vaughan (2009), Courtiel and Vaughan (2011), Kumar *et al*. (2015a, b). Seeing the application potential of Gerechte designs in forming efficient breeding designs, we have provided a series of Gerechte-based ME-PDCs and ME-CDCs. The obtained ME designs are very efficient, as the underlying Gerechte designs are also of high efficiency. In addition, whenever the proposed Gerechte designs exist, one can easily obtain these ME designs as a by-product.

Materials and Methods

Model and Experimental Setup

The statistical model for diallel crosses, arranged in a block setup, can be expressed as:

$$y_{ij} = \mu + g_i + g_j + s_{ij} + e_{ij} \quad (1)$$

where y_{ij} is the response from the cross between i^{th} line and j^{th} line; μ is the general mean effect; g_i and g_j are the gca effects of i^{th} line and j^{th} line, respectively; s_{ij} is the sca between i^{th} line and j^{th} line and e_{ij} is the error term independently and identically distributed having normal distributed with zero mean and constant variance. It may be noted here that $g_i + g_j + s_{ij}$ constitutes the cross effect.

Degree of Fractionation

Let N_{CDC} be the number of crosses involved in CDC designs, whereas N_{PDC} represents the number of crosses involved in PDC designs for a given number of lines v. The degree of fractionation, DF related to designs involving PDC is calculated as:

$$\text{DF} = \frac{N_{PDC}}{N_{CDC}} = \frac{2\,N_{PDC}}{v(v-1)}$$

Canonical Efficiency Factor

If $\mathbf{C}_d$ is the information matrix pertaining to the GCA effects of a complete block (orthogonal) design, d and C_d^* of the proposed incomplete block design, assuming equal estimated error variances, then the canonical efficiency factor (CEF) of the proposed design can be calculated as:

$$\text{CEF} = \frac{\text{Harmonic mean of non-zero eigenvalues of } C_d^*}{\text{Harmonic mean of non-zero eigenvalues of } \mathbf{C}_d}.$$

Results and Discussion

Method of Construction

In this section, we describe the construction methods of ME designs based on CDCs and PDCs using the specified Gerechte designs.

Method I

Firstly, an incomplete Gerechte design is constructed for an even number of lines, v ($\geq$ 8) and using that Gerechte design, one can easily obtain ME-PDCs with an excellent efficiency factor as well as a low degree of fractionation.

Construction of Gerechte Design

- Consider the initial row of elements as 1, v, v – 1, …, 2.
- Expand all odd columns cyclically and even columns reverse cyclically, up to $\left(\frac{v}{2}\right)^{the}$ element in each column.
- Take two adjacent columns from left to right to form a total of $\left(\frac{v}{2}\right)$ regions (sub-arrays).
- The resultant design will be an incomplete Gerechte design (G) with the number of rows (R) = number of regions (S) = number of replications (r) = $\left(\frac{v}{2}\right)$ and number of columns (C) = v.

Construction of ME-PDC

- Construct two sets from each of the regions of dimension $\left(\frac{v}{2}\right) \times 2$ such that the first set contains all the $[i, j]^{th}$ elements of a region where i = 1, 2, …, $\left(\frac{v}{2}\right)$ and j takes the value 1 (if i is odd) or 2 (if i is even). The second set contains complementary elements of the first set.
- Now construct a block of diallel crosses by making crosses within the sets by following the rule of moving forward, *i.e.*, cross the first and second elements first, for the second cross the second and third elements, and so on.
- Finally, a ME-PDC is obtained with $\left(\frac{v}{2}\right)$ blocks each of size v – 2.

Example: for v = 10, one can easily construct a Gerechte design using the above-mentioned steps, considering the initial row as 1, 10, 9, 8, 7, 6, 5, 4, 3, 2,1,10,...,3,2 (Table 1).

Table 1: Gerechte design for v = 10 with 5 regions (dim: 5 x 2)

	C1	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*	*C8*	*C9*	*C10*
R1	1	10	9	8	7	6	5	4	3	2
R2	2	9	10	7	8	5	6	3	4	1
R3	3	8	1	6	9	4	7	2	5	10
R4	4	7	2	5	10	3	8	1	6	9
R5	5	6	3	4	1	2	9	10	7	8

Consider the first region

1	10
2	9
3	8
4	7
5	6

First set will contain all the $[i, j]^{th}$ elements in this region where $i = 1, 2, \ldots, 5$. So, the elements of first set are {[1, 1], [2, 2], [3, 1], [4, 2], [5, 1]} $\Rightarrow$ {1, 9, 3, 7, 5} and its complementary elements for second set are {[1, 2], [2, 1], [3, 2], [4, 1], [5, 2]} $\Rightarrow$ {10, 2, 8, 4, 6}.

B1	1 × 9	9 × 3	3 × 7	7 × 5	10 × 2	2 × 8	8 × 4	4 × 6

From first set (1 × 9, 9 × 3, 3 × 7, 7 × 5) — From second set (10 × 2, 2 × 8, 8 × 4, 4 × 6)

Therefore, the final ME-PDC design can be obtained with 5 blocks of size 8 with CEF 0.7801 and DF = 0.4444 as shown in Table 2.

Method II

In this method, firstly, an incomplete Gerechte design is constructed for a composite number, $v = pq$; $p, q \geq 3$ and then taking crosses among inter-region elements, one can obtain ME-PDCs, which are suitable for breeding experiments.

Construction of Gerechte Design

- Write an array of order $p \times q$ such that it is in natural order and consider this as a base region.
- Rotate the columns of the base region, one cyclical shift at a time, and append vertically the base region to form q regions such that each column contains all the v symbols.
- Now, for each of the q regions, rotate the rows and append horizontally to form a complete Gerechte design.
- Delete the first row and the first column of each region.

Resultant arrangement is an incomplete Gerechte design (for $v = pq$) with $R = p(q - 1)$, $C = q(p - 1)$, $S = v = pq$ and $r = (p - 1) \times (q - 1)$.

Construction of ME-PDC

- Form q sets each constituting $\frac{S}{p-1}$ regions sharing the same rows
- Consider all possible unique pairs of regions within a set
- Consider $(q - 1)$ columns of the first region and $(q - 1)$ forward diagonals of the second region for each pair of regions
- Make $(p - 1)$ crosses between corresponding elements of $(q - 1)$ column-diagonal combinations (i^{th} column with i^{th} forward diagonal, $i = 1, 2, \ldots, q - 1$), constituting a block
- Proceeding in a similar manner, we get $\binom{p}{2}$ blocks from each set
- Repeat the process for all other sets, resulting into $q \times \binom{p}{2}$ blocks of size $(p - 1)(q - 1)$ each.

Example: For $v = 4 \times 4$, a Gerechte design of 16 regions of dimension 3×3 using the 2^{nd} associates of treatments constructed as follows as given in Table 3.

Let us consider two consecutive regions of the first row of regions, say,

6	7	8	10	11	12
10	11	12	14	15	16
14	15	16	2	3	4

Now, for i^{th} (i = 1, 2, 3) column of the first region, take elements diagonally mod$(v - 1)$ from the i^{th} element of the first sub-row of the second region, which is given as follows:

6	10	7	11	8	12
10	15	11	16	12	14
14	4	15	2	16	3

Table 2: ME - PDC for v = 10

B1	1 × 9	9 × 3	3 × 7	7 × 5	10 × 2	2 × 8	8 × 4	4 × 6
B2	9 × 7	7 × 1	1 × 5	5 × 3	8 × 10	10 × 6	6 × 2	2 × 4
B3	7 × 5	5 × 9	9 × 3	3 × 1	6 × 8	8 × 4	4 × 10	10 × 2
B4	5 × 3	3 × 7	7 × 1	1 × 9	4 × 6	6 × 2	2 × 8	8 × 10
B5	3 × 1	1 × 5	5 × 9	9 × 7	2 × 4	4 × 10	10 × 6	6 × 8

Remark: Two types of replications for the crosses can be observed in the designs developed through Method I.

Table 3: Gerechte design for v = 16 with 16 regions (dim: 3 x 3)

	C1	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*	*C8*	*C9*	*C10*	*C11*	*C12*
R1	6	7	8	10	11	12	14	15	16	2	3	4
R2	10	11	12	14	15	16	2	3	4	6	7	8
R3	14	15	16	2	3	4	6	7	8	10	11	12
R4	7	8	5	11	12	9	15	16	13	3	4	1
R5	11	12	9	15	16	13	3	4	1	7	8	5
R6	15	16	13	3	4	1	7	8	5	11	12	9
R7	8	5	6	12	9	10	16	13	14	4	1	2
R8	12	9	10	16	13	14	4	1	2	8	5	6
R9	16	13	14	4	1	2	8	5	6	12	9	10
R10	5	6	7	9	10	11	13	14	15	1	2	3
R11	9	10	11	13	14	15	1	2	3	5	6	7
R12	13	14	15	1	2	3	5	6	7	9	10	11

Table 4: ME-PDC for v = 16

B1	6 × 15	10 × 15	14 × 4	7 × 11	11 × 16	15 × 2	8 × 12	12 × 14	16 × 3
B2	6 × 14	10 × 3	14 × 8	7 × 15	11 × 4	15 × 6	8 × 16	12 × 2	16 × 7
B3	6 × 2	10 × 7	14 × 12	7 × 3	11 × 8	15 × 10	8 × 4	12 × 6	16 × 11
B4	10 × 14	14 × 3	2 × 8	11 × 15	15 × 4	3 × 6	12 × 16	16 × 2	4 × 7
B5	10 × 2	14 × 7	2 × 12	11 × 3	15 × 8	3 × 10	12 × 4	16 × 6	4 × 11
B6	14 × 2	2 × 7	6 × 12	15 × 3	3 × 8	7 × 10	16 × 4	4 × 6	8 × 11
B7	7 × 11	11 × 16	15 × 1	8 × 12	12 × 13	16 × 3	5 × 9	9 × 15	13 × 4
B8	7 × 15	11 × 4	15 × 5	8 × 16	12 × 1	16 × 7	5 × 13	9 × 3	13 × 8
B9	7 × 3	11 × 8	15 × 9	8 × 4	12 × 5	16 × 11	5 × 1	9 × 7	13 × 12
B10	11 × 15	15 × 4	3 × 5	12 × 16	16 × 1	4 × 7	9 × 13	13 × 3	1 × 8
B11	11 × 3	15 × 8	3 × 9	12 × 4	16 × 5	4 × 11	9 × 1	13 × 7	1 × 12
B12	15 × 3	3 × 8	7 × 9	16 × 4	4 × 5	8 × 11	13 × 1	1 × 7	5 × 12
B13	8 × 12	12 × 13	16 × 2	5 × 9	9 × 14	13 × 4	6 × 10	10 × 16	14 × 1
B14	8 × 16	12 × 1	16 × 6	5 × 13	9 × 2	13 × 8	6 × 14	10 × 4	14 × 5
B15	8 × 4	12 × 5	16 × 10	5 × 1	9 × 6	13 × 12	6 × 2	10 × 8	14 × 9
B16	12 × 16	16 × 1	4 × 6	9 × 13	13 × 2	1 × 8	10 × 14	14 × 4	2 × 5
B17	12 × 4	16 × 5	4 × 10	9 × 1	13 × 6	1 × 12	10 × 2	14 × 8	2 × 9
B18	16 × 4	4 × 5	8 × 10	13 × 1	1 × 6	5 × 12	14 × 2	2 × 8	6 × 9
B19	5 × 9	9 × 14	13 × 3	6 × 10	10 × 15	14 × 1	7 × 11	11 × 13	15 × 2
B20	5 × 13	9 × 2	13 × 7	6 × 14	10 × 3	14 × 5	7 × 15	11 × 1	15 × 6
B21	5 × 1	9 × 6	13 × 11	6 × 2	10 × 7	14 × 9	7 × 3	11 × 5	15 × 10
B22	9 × 13	13 × 2	1 × 7	10 × 14	14 × 3	2 × 5	11 × 15	15 × 1	3 × 6
B23	9 × 1	13 × 6	1 × 11	10 × 2	17 × 7	2 × 9	11 × 3	15 × 5	3 × 10
B24	13 × 1	1 × 6	5 × 11	14 × 2	2 × 7	6 × 9	15 × 3	3 × 5	7 × 10

Remark: Three types of replications for the crosses can be seen in all the designs obtained from Method II.

Table 5: Gerechte design for v = 10 with 5 regions (dim: 5 x 2)

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
R1	1	10	9	8	7	6	5	4	3	2
R2	2	9	10	7	8	5	6	3	4	1
R3	3	8	1	6	9	4	7	2	5	10
R4	4	7	2	5	10	3	8	1	6	9
R5	5	6	3	4	1	2	9	10	7	8

These are the 9 crosses obtained for the first block. Finally, the ME-PDC design can be obtained with 24 blocks of size 9 with CEF 0.8228 and DF = 0.7 as shown in Table 4.

Method III

Consider again the Gerechte designs obtained in Method I. Highly efficient ME-CDCs (if $\frac{v}{2}$ is odd) can be obtained using the following simple steps, for the number of lines, *v* ($\geq$ 8).

Construction of ME-CDC

- Take all unique pairs of regions.
- Within a pair of regions, form two blocks such that the first set of *v* crosses are obtained by crossing the two lines appearing on the same position and the second set of crosses are obtained directly by crossing the two lines in the same row within each region.
- Repeat the process for all the $\binom{v/2}{2}$ pairs of regions, giving rise to $\frac{v}{2}(\frac{v}{2}-1)$ blocks of size *v*.

Example: For *v* = 8, a Gerechte design of 4 regions of dimension 4 × 2 constructed as follows as given in Table 5.

Using this Gerechte design, we obtain all the crosses block-wise, as discussed above. The resultant design, ME-PDC with 20 blocks each of size 10, and CEF 0.887, is given in Table 6.

Illustration of Analysis Using Hypothetical Data

A hypothetical data set for ME-PDC design for a number of lines 10 in 5 blocks of size 10 each, is shown in Table 7.

The above dataset has been analyzed using model (1), and the results are consolidated in the following Tables 8 and 9).

R-package

The R package, "MEDesigns", is a specialized tool designed for the generation of proposed ME designs and analysis of ME designs. These designs are very useful in agricultural and biological research, particularly for experiments involving genetic crosses. The package contains functions that incorporate unique algorithms to generate our

Table 6: ME-CDC for v = 10

B1	1 × 10	2 × 9	3 × 8	4 × 7	5 × 6	9 × 8	10 × 7	1 × 6	2 × 5	3 × 4
B2	1 × 10	2 × 9	3 × 8	4 × 7	5 × 6	7 × 6	8 × 5	9 × 4	10 × 3	1 × 2
B3	1 × 10	2 × 9	3 × 8	4 × 7	5 × 6	5 × 4	6 × 3	7 × 2	8 × 1	9 × 10
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
B18	7 × 5	8 × 6	9 × 7	10 × 8	1 × 9	6 × 4	5 × 3	4 × 2	3 × 1	2 × 10
B19	7 × 3	8 × 4	9 × 5	10 × 6	1 × 7	6 × 2	5 × 1	4 × 10	3 × 9	2 × 8
B20	5 × 3	6 × 4	7 × 5	8 × 6	9 × 7	4 × 2	3 × 1	2 × 10	1 × 9	10 × 8

Remark: Two types of replications for the crosses can be found in all the proposed designs obtained from Method III.

Table 7: Illustration of analysis using hypothetical data

B1	1 × 9 (129.50)	9 × 3 (60.83)	3 × 7 (120.99)	7 × 5 (68.31)	10 × 2 (69.66)	2 × 8 (75.73)	8 × 4 (78.23)	4 × 6 (80.03)
B2	9 × 7 (116.23)	7 × 1 (119.67)	1 × 5 (127.01)	5 × 3 (98.83)	8 × 10 (117.14)	10 × 6 (114.67)	6 × 2 (109.80)	2 × 4 (103.83)
B3	7 × 5 (100.10)	5 × 9 (94.86)	9 × 3 (86.63)	3 × 1 (131.19)	6 × 8 (88.42)	8 × 4 (108.72)	4 × 10 (94.90)	10 × 2 (95.17)
B4	5 × 3 (83.98)	3 × 7 (122.37)	7 × 1 (122.12)	1 × 9 (134.00)	4 × 6 (85.39)	6 × 2 (97.17)	2 × 8 (101.57)	8 × 10 (93.96)
B5	3 × 1 (110.85)	1 × 5 (127.41)	5 × 9 (114.39)	9 × 7 (106.75)	2 × 4 (91.62)	4 × 10 (84.81)	10 × 6 (90.97)	6 × 8 (82.76)

Response for each cross is given in parentheses.

Table 8: ANOVA

SV	*df*	*SS*	*MSS*	*F-value*	*Pr(>F)*
Block	4	3315	828.70	12.85	<.0001
Cross	19	9746	512.90	7.95	<.0001
Error	16	1032	64.50		

GCA and SCA sum of squares have now been calculated from the cross sum of squares after necessary adjustments.

Table 9: GCA and SCA Analysis

SV	*df*	*SS*	*MSS*	*F-value*	*Pr(>F)*
gca	9	5830.42	647.82	10.04	<.0001
sca	10	3582.73	358.27	5.55	0.001

Remark: It may be noted here that all sca effects are not estimable. However, the estimable sca effects are highly significant.

Table 10: List of ME designs

v	*m*	*n*	*b*	*k*	*CEF*	*DF*	*Method*
8	-	-	4	6	0.683	0.429	I
9	3	3	9	4	0.628	0.750	II
10	-	-	5	8	0.780	0.444	I
	-	-	20	10	0.887	1	III
12	-	-	6	10		0.455	I
	4	3	18	6	0.763	0.818	II
	3	4	12	6	0.696	0.545	II
14	-	-	7	12	0.860	0.462	I
	-	-	42	14	0.923	1	III
15	3	5	15	8	0.721	0.429	II
	5	3	30	8	0.824	0.857	II
16	-	-	8	14	0.882	0.467	I
	4	4	24	9	0.823	0.700	II
18	-	-	9	16	0.898	0.471	I
	3	6	18	10	0.737	0.354	II
	6	3	45	10	0.863	0.882	II
	-	-	72	18	0.941	1	III
20	-	-	10	18	0.910	0.474	I
	4	5	30	12	0.864	0.632	II
	5	4	40	12	0.877	0.758	II
21	3	7	21	12	0.747	0.300	II
	7	3	63	12	0.882	0.900	II
22	-	-	11	20	0.920	0.476	I
	-	-	110	22	0.952	1	III
24	-	-	12	22	0.927	0.478	I
	3	8	24	14	0.754	0.261	II
	8	3	84	14	0.902	0.913	II
	4	6	36	15	0.880	0.587	II
	6	4	60	15	0.907	0.841	II
25	5	5	50	16	0.902	0.733	II
26	-	-	13	24	0.934	0.480	I
	-	-	156	26	0.96	1	III
27	3	9	27	16	0.759	0.231	II
	9	3	108	16	0.911	0.923	II
28	-	-	14	26	0.939	0.481	I
	4	7	42	18	0.885	0.500	II
	7	4	84	18	0.924	0.889	II
30	-	-	15	28	0.944	0.483	I
	3	10	30	18	0.763	0.207	II
	10	3	135	18	0.924	0.931	II
	5	6	60	20	0.918	0.648	II
	6	5	75	20	0.926	0.770	II
	-	-	210	30	0.965	1	III

It can be seen from the above table that the proposed designs are highly efficient with low degree of fractionation in most cases. Obviously, for ME-CDC, degree of fractionation is exactly 1.

proposed designs along with their parameters, eigenvalues, degree of fractionation, and CEF. Also, another function has been developed for studying the properties of a given ME-PDC. In addition, the most important function is developed for data analysis, which may be very useful for the experimenters. "MEDesigns" is particularly suited for researchers in agricultural statistics, plant breeding, and other related fields. Simplifying the complex task of generating mating environmental designs enhances the applicability of these designs. The package is freely available on the Comprehensive R Archive Network (CRAN) (https://cran.r-project.org/package=MEDesigns).

Using this R package, CEFs of the designs are computed and listed along with the parameters of ME designs obtained using Method I, II and III for $v \leq 30$ (see Annexure) (Table 10).

Discussion

Considering a part of or a full Gerechte design as a base design and moving towards ME designs for a diallel cross experiment is a unique approach. One obvious result is that whenever these Gerechte designs exist, one can easily obtain ME-PDCs and ME-CDCs through our proposed methods. These Gerechte designs exist for a wide range of parameters, and the developed designs are highly efficient. Furthermore, Table 1 contains a list of designs to provide a comprehensive overview of the proposed designs. To increase the utility of this research work, an R package has also been developed to provide easy accessibility to the designs along with the analysis of the provided data.

Conflict of interest

The authors declare that there is no conflict of interest.

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